Let X1, …..., Xn be independent and identically U[0,] distributed random variables. Show that

is asymptotically unbiased and consistent for γ(θ) = θe-1.

**Solution:**

Since *X1, X2, …..., Xn*

Is the Dirac function, whose value is 1 if *x*  and otherwise *0*. Then we have jointly density function:

Compute the expectation of :

Asymptotic unbiasedness means that the estimator’s bias decrease until it reaches zero, as n → ∞, implying that the estimator’s expected value converges the right value of the parameter [1].

𝜃

Now, from (1)

So,

Take the limit (Both numerator and denominator have limits and :

Consistent means that for high sample sizes as n → ∞, the likelihood that deviates from the real, but unknown, value becomes negligible [1].

So,

**Reference**:

1. https://stats.stackexchange.com/questions/280684/intuitive-understanding-of-the-difference-between-consistent-and-asymptotically